

# Physical Chemistry of Colloids



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**Lecture 3, March 6, 2019**

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# Previously in ColloidsPhysChem...(I)

a fluid interface contracts → *minimum area*

zero-thickness membrane model to quantify  
"contractile tendency" of fluid interfaces

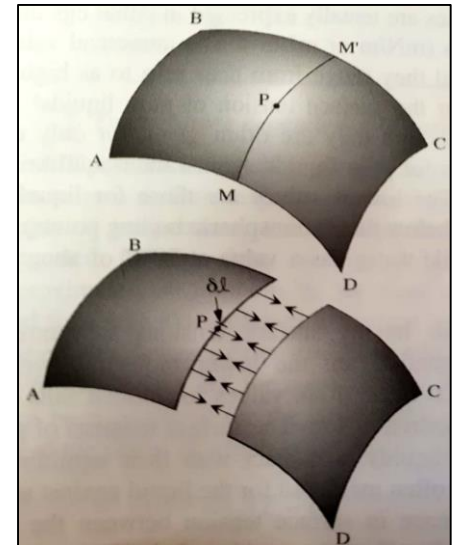
**interfacial tension**

force/length along the dividing line @ P

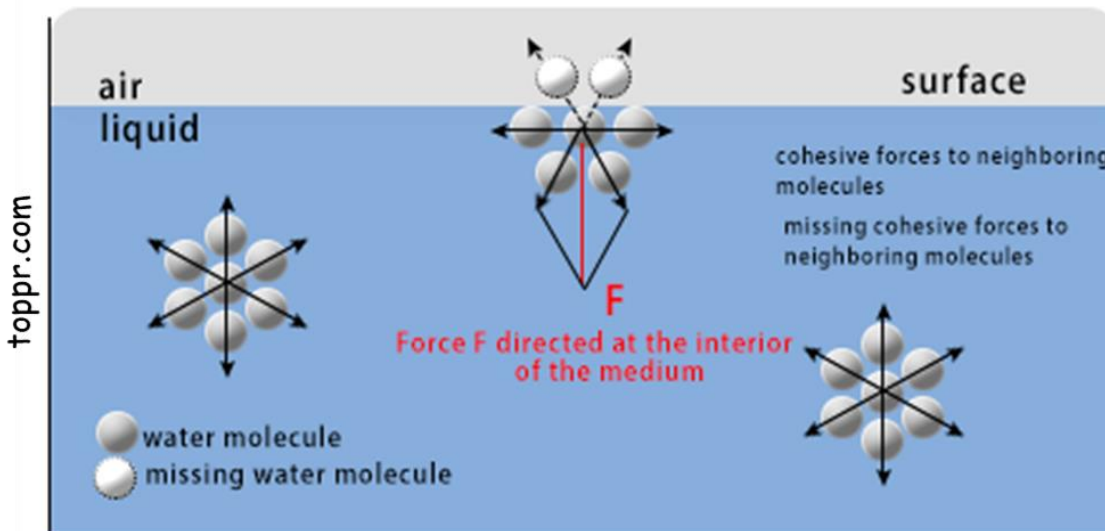
$$\sigma = \frac{d|F|}{dl}$$

$$\delta W = \sigma dA$$

unit (SI): N/m or J/m<sup>2</sup>  
more common: mN/m



Interfacial tension may be interpreted in terms of *unbalanced intermolecular forces*



→ net inward attraction  
pulls surface molecules  
toward the interior; surface  
tries to minimize its area

# Previously in ColloidsPhysChem...(II)

## Attractive vdW interactions

$\delta^+$  permanent  $\delta^-$

$\delta^+$  permanent  $\delta^-$

$$V_{Keesom} = -\frac{B_{polar}}{r^6}$$

~ 5-25 kJ/mol

$\delta^+$  permanent  $\delta^-$

$\delta^+$  induced  $\delta^-$

$$V_{Debye} = -\frac{B_{induced}}{r^6}$$

~ 2-10 kJ/mol

$\delta^+$  induced  $\delta^-$

$\delta^+$  induced  $\delta^-$

$$V_{London} = -\frac{B_{disp}}{r^6}$$

~ 0.05-40 kJ/mol

( $k_B T @ 25^\circ C = 2.47 \text{ kJ/mol}$ )

All attractive vdW interactions vary as  $1/r^6$

$$V_{attractive} = -\frac{B_{attractive}}{r^6}$$

## Repulsive vdW interactions

- strong repulsion when electron clouds overlap (hard-core repulsion)
- manifestation of Pauli's exclusion principle

$$V_{repulsive} = \frac{B_{repulsive}}{r^{12}}$$

# Previously in ColloidsPhysChem...(III)

Total interaction between two molecules:  $V_{\text{attractive}} + V_{\text{repulsive}}$

$$V_{\text{tot}} = 4\epsilon \left[ \left( \frac{\delta}{r} \right)^{12} - \left( \frac{\delta}{r} \right)^6 \right]$$

$\delta$ :  $r$  for which  $V_{\text{tot}} = 0$   
(molecular diameter)

$\epsilon$ : depth of potential well

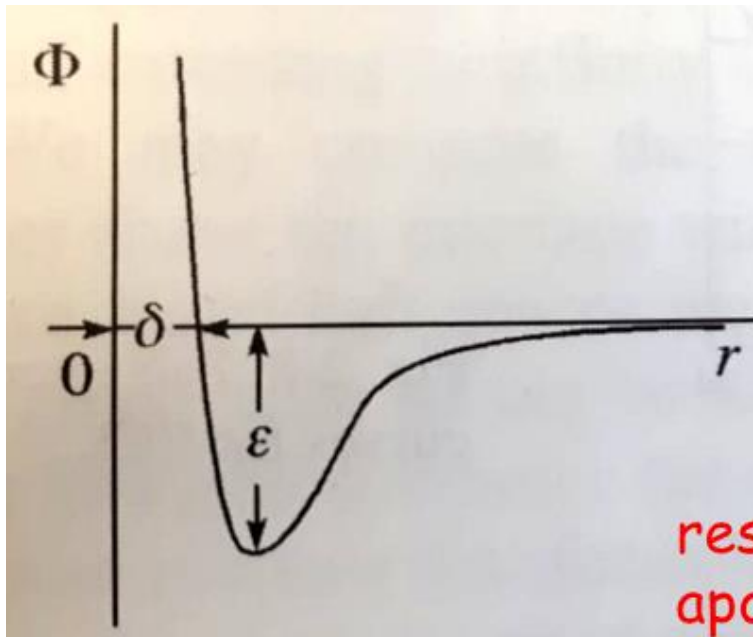


Fig. 2-7: Schematic of Lennard-Jones potential function.

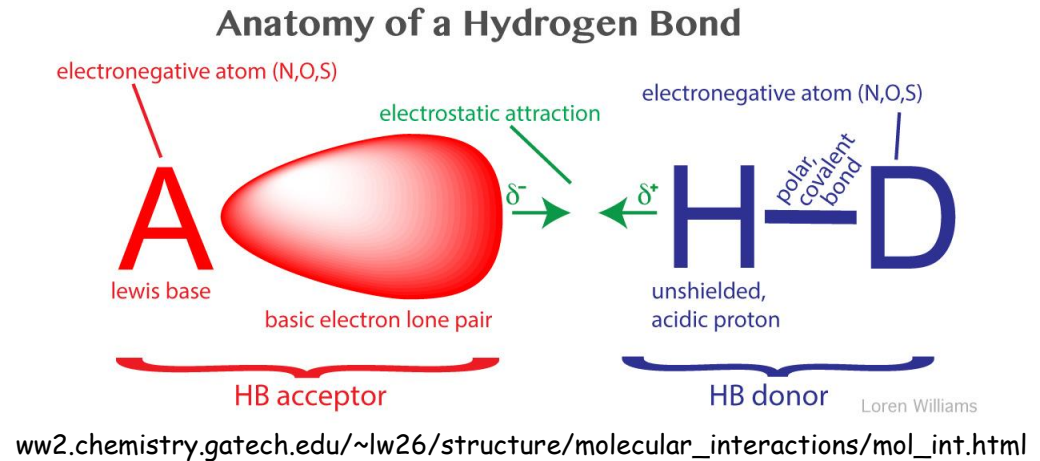
restricted to approx. spherical, apolar or weakly polar molecules

# Previously in ColloidsPhysChem...(IV)

typically written:  $D-H \cdots A$

**D**: H-bond donor (N, O, F, S)

**A**: H-bond acceptor (N, O, F, S)  
possesses a lone pair of  $e^-$



**water: the perfect example of H-bonding**

- each water molecule can form 4 H-bonds  
with surrounding water molecules

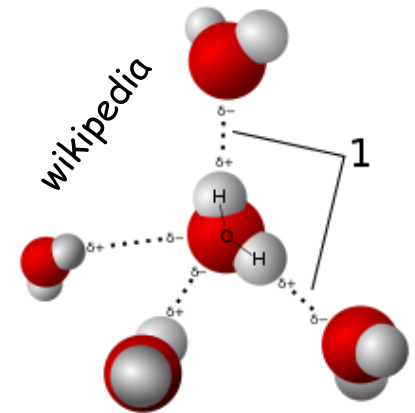
- 2 lone  $e^-$  pairs + 2  $\delta^+$  H

- reason for **very high  $T_{\text{boil}}$**

- **directional** & with **limited number of interaction partners**

- strong; typical energies  $\sim 5\text{-}30 \text{ kJ/mol}$  ( $k_B T @ 25 \text{ }^\circ\text{C} = 2.47 \text{ kJ/mol}$ )

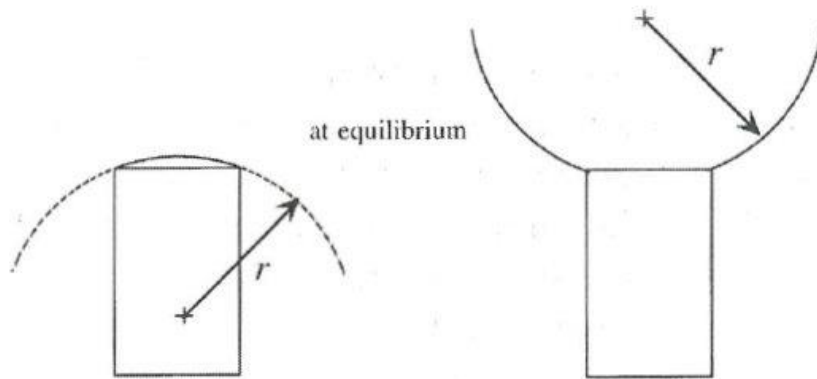
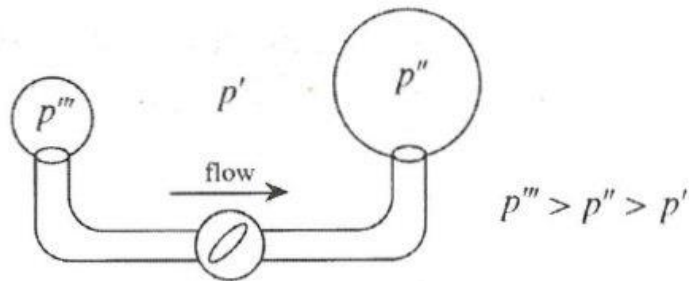
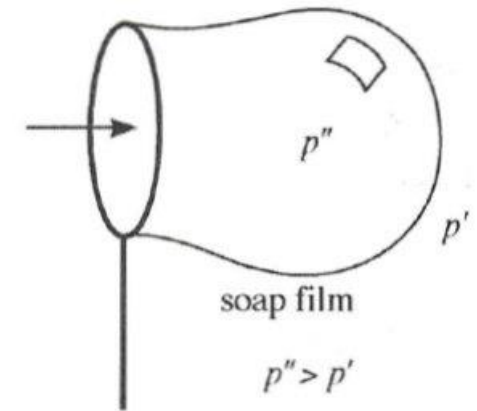
$\rightarrow$  **dynamically formed & broken**



# Pressure difference across curved interfaces

## Defining the *shape & location* of fluid interfaces

- fundamental problem of capillary hydrostatics
- basis for most methods for measuring  $\sigma$
- affects motion of liquids in porous media, thermodyn. properties of bubbles & drops



## Deformation of an elastic membrane

- $p''$  (concave side)  $>$   $p'$  (convex side)
- $\Delta p$  proportional to the curvature
- larger bubble grows @ the expense of the smaller one
- flow continues until curvatures are equal

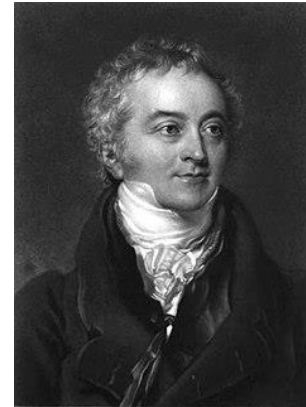
Fig. 2-15: Spontaneous flow occurs from the smaller bubble (higher curvature) to the larger bubble until the spherical cap at the location where the smaller bubble started has the same radius of curvature as the final larger bubble.

# Young-Laplace equation

Pierre-Simon Laplace  
(1749-1827)



Thomas Young  
(1773 - 1829)



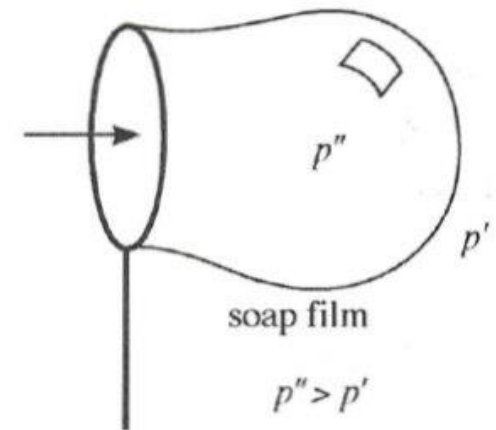
wikipedia

$$\Delta p = p'' - p' = \sigma \kappa$$

$\Delta p$ : pressure jump across  
a curved fluid interface

$\sigma$ : interfacial tension

$\kappa$ : local curvature of the interface



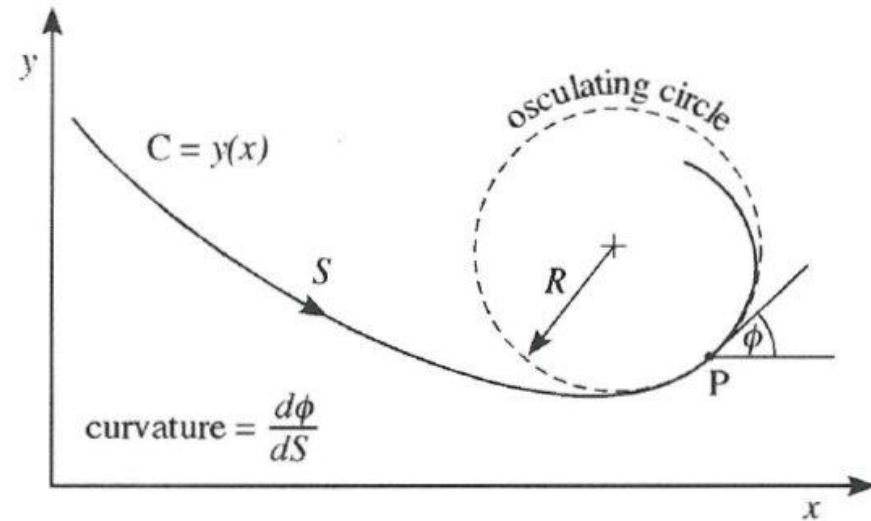
# The curvature of a surface

**Curvature of a plane curve  $y(x)$  @  $P$**   
 rate of change of direction ( $\phi$ ) with arc length ( $S$ )

$$\kappa = \frac{d\phi}{dS} = \pm \frac{d^2y}{dx^2} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{-\frac{3}{2}}$$

for a circle ( $\emptyset 2R$ ):  $\kappa = \pm \frac{1}{R}$

radius of circle of curvature @  $P$ : **radius of curvature** of  $y(x)$  @  $P$



## Curvature of a surface

two orthogonal planes passing through normal @ POI

planes cut surface in two plane curves

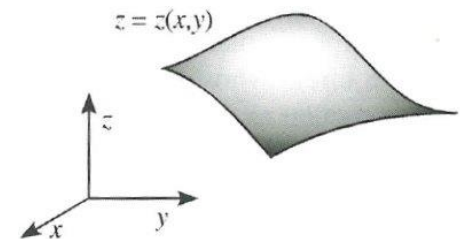
$$\kappa = \pm \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$R_1, R_2$ : principal radii of curvature

## General expression

$$\kappa = \pm \frac{\left( \frac{\partial^2 z}{\partial x^2} \right) \left[ 1 + \left( \frac{\partial z}{\partial y} \right)^2 \right] - 2 \left( \frac{\partial z}{\partial x} \right) \left( \frac{\partial z}{\partial y} \right) \left( \frac{\partial^2 z}{\partial x \partial y} \right) + \left( \frac{\partial^2 z}{\partial y^2} \right) \left[ 1 + \left( \frac{\partial z}{\partial x} \right)^2 \right]}{\left[ 1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \right]^{3/2}}$$

$z = z(x, y)$



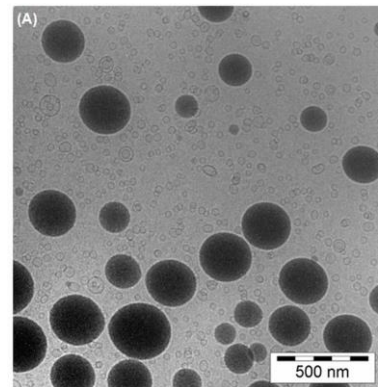
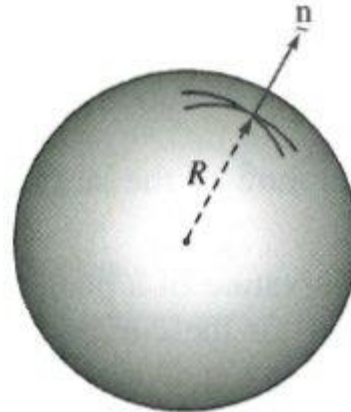


# Curvatures of geometries of special interest

many surfaces of interest possess certain symmetries  
 → simplified expressions for  $\kappa$

**Spheres or spherical caps**  
 any normal to the surface  
 passes through center

$$\kappa = \pm \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{2}{R}$$



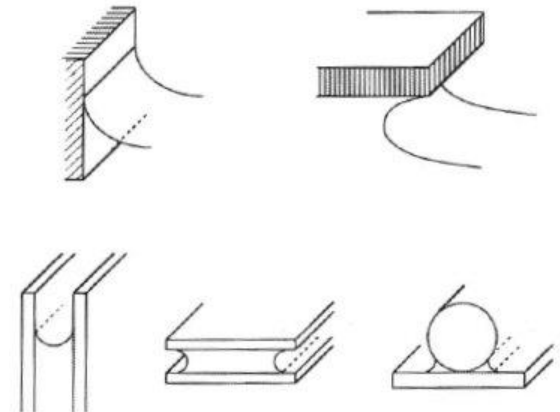
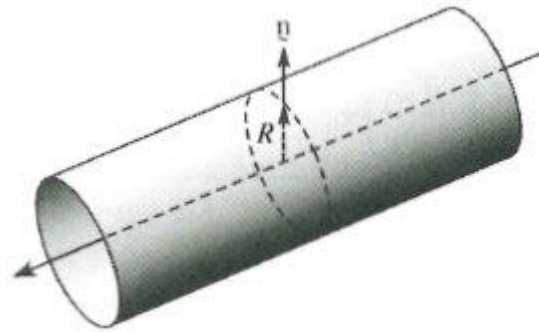
PLoS ONE 2015, 10, e0130674

**Cylindrical surfaces**

any normal to the surface  
 passes through & is normal  
 to the axis of the cylinder

1<sup>st</sup> plane: circle with  $R$   
 2<sup>nd</sup> plane: rectangle

$$\kappa = \pm \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{R}$$



# Derivation of Young-Laplace equation

small patch (ADBC) of surface centered @ P  
& enclosed by a curve @ a distance  $\rho$  from P

$\rho$  is measured along the surface & is taken to be *very small*

phase (") is on the *lower (concave) side* of the patch; phase (') is on the *upper (convex) side*

orthogonal lines AB and CD made by a pair of orthogonal planes passing through  $\hat{n}$

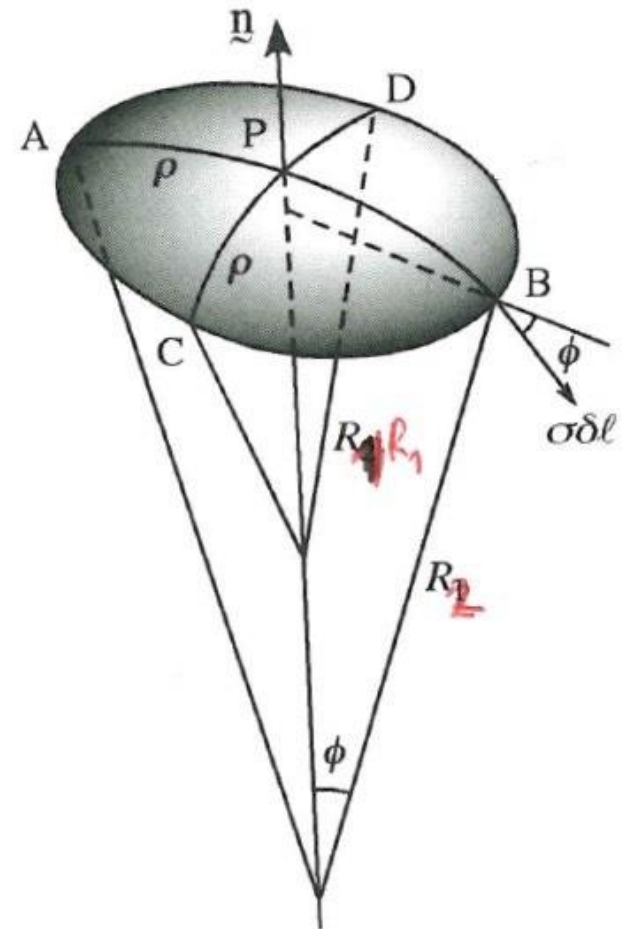
net upward (+ $\hat{n}$  direction) force on the patch

$$F \uparrow = (p'' - p')\pi\rho^2$$

net downward (- $\hat{n}$  direction) force on the patch

$$F \downarrow = -\sigma\pi\rho^2 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$F \downarrow = F \uparrow \quad \Rightarrow \quad \Delta p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$



# Methods for measuring surface tension

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**Basis:** solutions to the Young-Laplace (Y-L)

## **Geometric methods**

- based on **direct determination of interface shape or position**
- finding a  $\sigma$  value that produces the best match between a measured **interfacial profile & a solution of the Y-L eq.**

## **Force methods**

- based on the **measurement of a force (or mass, volume, pressure) & its comparison with the value computed** from the Y-L eq.
- usually solid object is suspended in or detached from a fluid interface or drop/bubble detached from an orifice
- interfaces of **high degree of symmetry** used (axisymmetric/cylindrical)

## ***Geometric methods***

- capillary rise
- sessile drop (captive bubble)
- pendant drop (pendant bubble)
- spinning drop
- oscillating jet
- contracting circular jet

## ***Force methods***

- du Noüy ring detachment
- Wilhelmy plate (or rod)
- Langmuir barrier
- drop weight (volume)
- maximum bubble pressure



# Coffee break

*Why Mario must add soap to water to make bubbles?  
How large can the bubbles be?*

# Capillary rise

determination of meniscus position of a liquid in a tube ( $\emptyset r$ )

$\theta < 90^\circ$

→ concave meniscus

→  $p_{up} > p_{down}$

equilibrium

→ meniscus rises until  $p_{down}$  can support the meniscus curvature

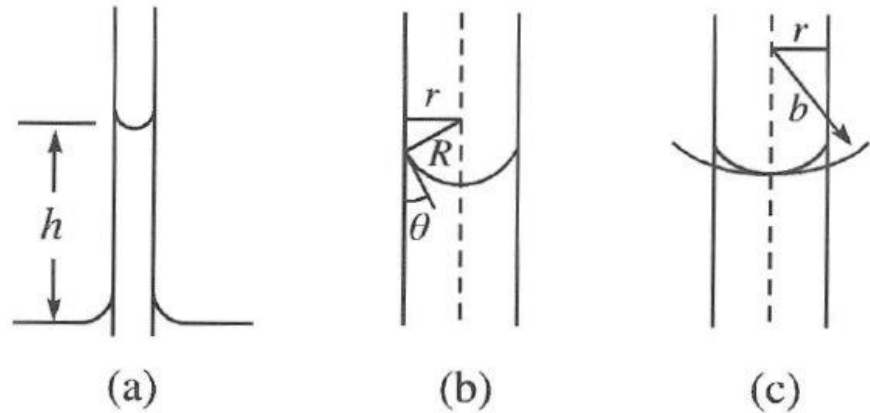


Fig. 2-38: The capillary rise method for measuring surface tension. (a)  $h$  is the equilibrium rise height, (b) If the meniscus is spherical, but the contact angle is  $> 0^\circ$ , the radius of the meniscus is  $R = r/\cos\theta$ , where  $r$  is the radius of the capillary, and  $\theta$  is the contact angle, (c) if the meniscus is flattened by gravity, the radius of the curvature at its apex is  $b > r$ .

for *small*  $r$  &  $\theta = 0^\circ$  gravity does not deform the meniscus

$$\Delta p = p_{up} - p_{down} = p_o - (p_o - \rho gh) = \rho gh$$

$$\Delta p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{2\sigma}{r}$$

$$\left. \begin{array}{l} \rho gh \\ \frac{2\sigma}{r} \end{array} \right\} \sigma = \frac{\rho grh}{2}$$

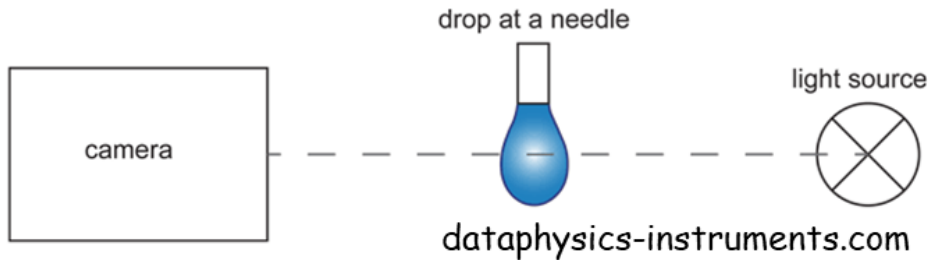
$h$ : eq. rise height

$\rho$ : liquid density

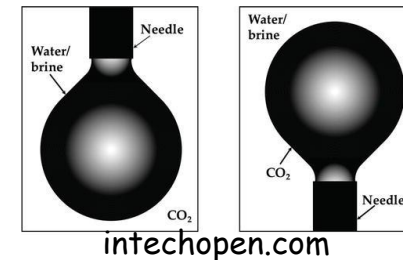
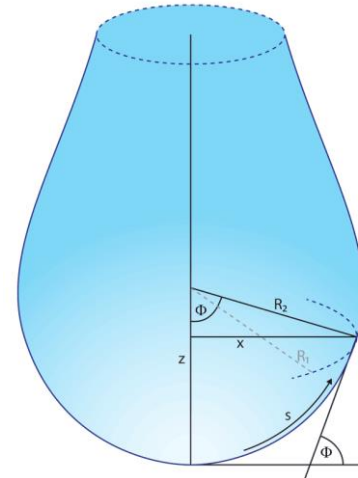
$g$ : grav. acceleration

# Pendant drops & captive bubbles

**Pendant drop (captive bubble):** a drop (bubble) suspended from a needle ( $\varnothing 2r$ ) in a bulk fluid phase



optical system used to take an **image of the drop**  
software performs **drop shape analysis**



**Drop shape:** determined by **interfacial tension - gravity** competition

$$\Delta P(z) = \Delta P_0 \pm \Delta \rho g z$$

$$\Delta P = (P_{\text{int}} - P_{\text{ext}}) = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_1} + \frac{\sin \Phi}{x} = \frac{2}{R} \pm \frac{\Delta \rho g z}{\sigma}$$

$$\frac{d\Phi}{ds} = -\frac{\sin \Phi}{x} + \frac{2}{R} \pm \frac{\Delta \rho g z}{\sigma}$$

$$\frac{dx}{ds} = \cos \Phi$$

$$\frac{dz}{ds} = \sin \Phi$$

$$0 = x(s=0) = z(s=0) = \Phi(s=0)$$

numerical fit of  
theor. drop shape  
to exp. shape  $\rightarrow \sigma$

# du Noüy ring & Wilhelmy plate

measurement of force required to detach a (Pt) ring ( $\varnothing 2r$ ) from a liquid surface

maximum in measured force against height of ring above undisturbed surface

beyond max. force  
 → meniscus contracts before final detachment

max. downward force  $F \downarrow = \frac{4\pi r \sigma}{F_c}$

$r$ : ring radius       $F_c$ : correction factor

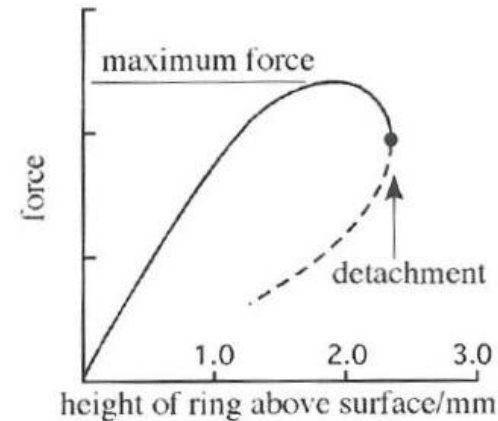
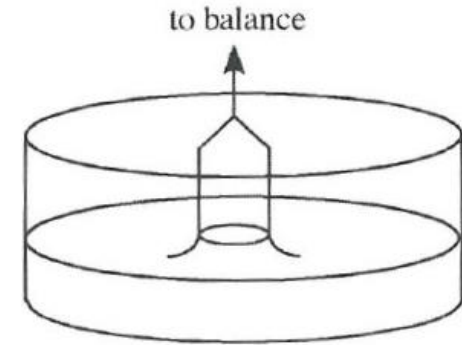


Fig. 2-40: du Noüy ring detachment method.

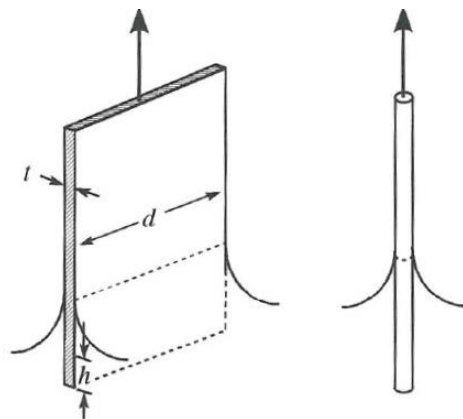


Fig. 2-41: Wilhelmy slide (or rod).

measurement of downward force on an partially immersed object

$$F_{net} = \sigma(2d + 2t) - (\rho g h t d)$$

# Maximum bubble pressure

formation of a bubble from a nozzle tip ( $\varnothing 2r$ )

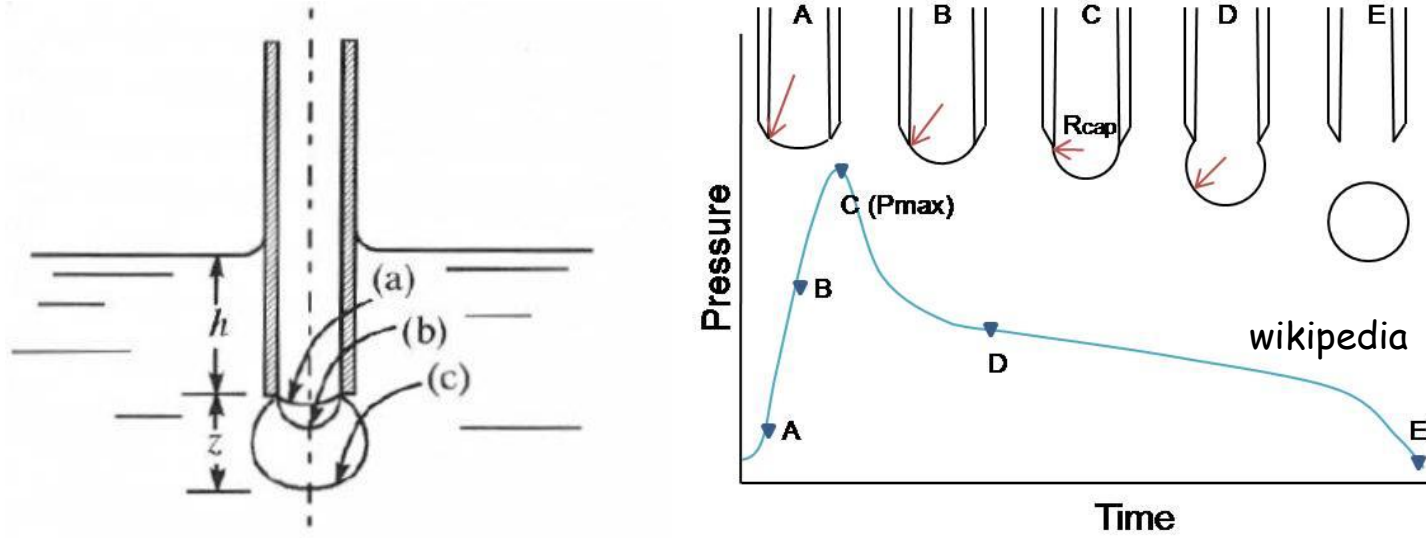


Fig. 2-46: Maximum bubble pressure method.

$P_{max}$  required when **radius of curvature** of the bubble is **minimum**

hemispherical surface  $\rightarrow P_{max} - P_{liq} = \frac{2\sigma}{r}$

## advantages

- **very rapid** method (ms or less); can measure **dynamic  $\sigma$**
- formed **surface is fresh**



# The hydrophobic effect (I)

## Hydrophobic interaction (IUPAC)

Tendency of **hydrocarbons** (or of lipophilic hydrocarbon-like groups in solutes) to **form intermolecular aggregates in an aqueous medium**, & analogous intramolecular interactions.

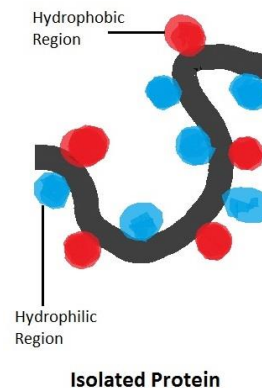
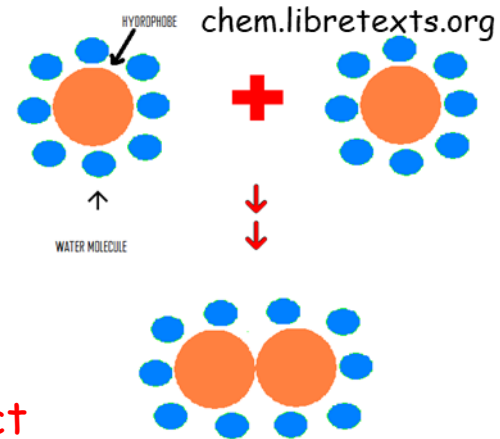
The name arises from the attribution of the phenomenon to the **apparent repulsion between water & hydrocarbons**.

However, the phenomenon ought to be attributed to the **effect of the hydrocarbon-like groups on the water-water interaction**.

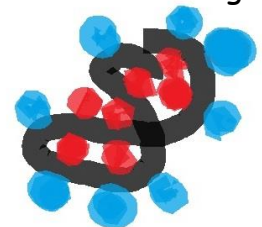
responsible for:

- the **separation of oil-water mixtures** into the two components
- **biological effects**: cell membrane & vesicle formation, protein folding, insertion of membrane proteins into the nonpolar lipid environment & protein-small molecule associations

**hydrophobes**: substances for which the hydrophobic eff. is observed



chem.libretexts.org



Protein in aqueous solution

# The hydrophobic effect (II)

thermodynamic factors giving rise to the hydrophobic effect: complex & still **not completely understood**

$$\Delta G = \Delta H - T \Delta S$$

## Small hydrophobic inclusions

- number of **H-bonds retained** but **degree of freedom** in how to form them **reduces**
- "**structuring**" of **water molecules** around the inclusion  
→ **S is reduced**,  $\Delta G > 0$

## Large (>1 nm) hydrophobic inclusions

- on average: one H-bond ( $\sim 10 k_B T$ ) per water molecule broken
- replacement of H-bond with **vdW interaction** (lower inter. energy)

*Nature* 2005, 437, 640

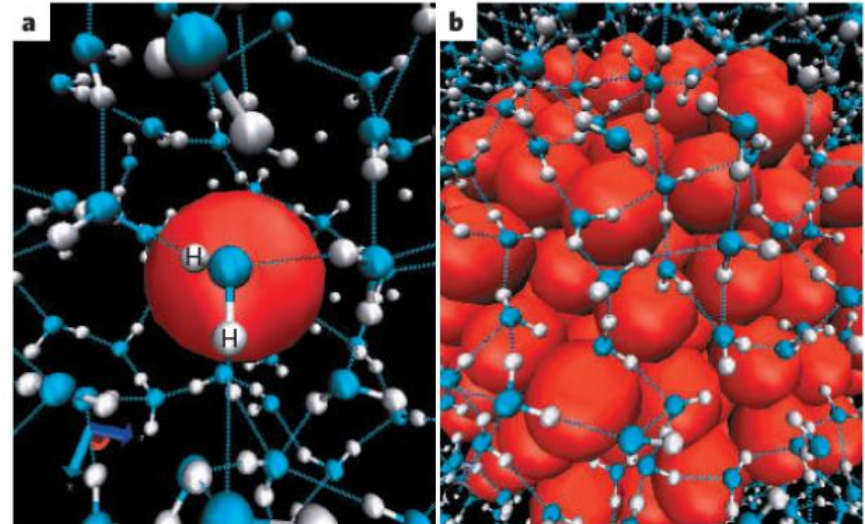


Figure 1 | Configurations of liquid water molecules near hydrophobic cavities in molecular-dynamics simulations.

## High hydrophobe concentration

- **segregation** of water & hydrophobic species (phase separation)

# Surfactants or amphiphiles (I)

**surfactant:** short for **surface active agent** (Langmuir); a substance that is surface-active: **adsorbs at an interface & reduces  $\sigma$**

**amphiphile:** a substance that has at least two portions segregated from one another, one being hydrophilic (head) & the other hydrophobic (tail)  
(*αμφί*: "from both sides"; *φίλος*: "friend")

**lipid:** long chain aliphatic hydrocarbons (fats) & derivatives originating in living cells; **only some**, such as **fatty acids**, are also **surfactants**

**detergent:** generally refers to **synthetic surfactants** used in cleaning agents

often classified according to head group

Anionic	Cationic	Non-ionic	Zwitterionic
SDBS	CTAB	D-123	DMBC

**ionic**

**non-ionic**

(C chain in head group interrupted by O atoms → polarity)

**zwitterionic**

low pH: cationic

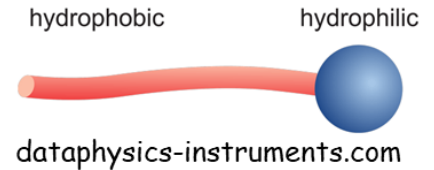
high pH: anionic

Table 3-3: Some typical surfactant head groups

-OH	Hydroxyl
-COOH (low pH)	Carboxyl
-COO <sup>-</sup> (high pH)	Carboxylate
-SO <sub>4</sub> <sup>-</sup>	Sulfate
-SO <sub>3</sub> <sup>-</sup>	Sulfonate
-H <sub>2</sub> PO <sub>4</sub> <sup>-</sup> (mod. pH)	Phosphate
-NH <sub>2</sub> (high pH)	Amino
-NH <sub>3</sub> <sup>+</sup> (low pH)	Ammonium
-N(CH <sub>3</sub> ) <sub>3</sub> <sup>+</sup>	Trimethylammonium, or Quaternary ammonium
-(OCH <sub>2</sub> CH <sub>2</sub> )OH	Polyoxyethylene, or polyethylene oxide

# Surfactants (II)

key feature of surfactants: **segregation** of hydrophilic & hydrophobic moieties



## hydrophilic head

can form **H-bonds** with water

→ "immerses" in aqueous phases

## hydrophobic tail

cannot form **H-bonds** with water

→ "immerses" in oil phases or "sticks" to hydrophobic solids

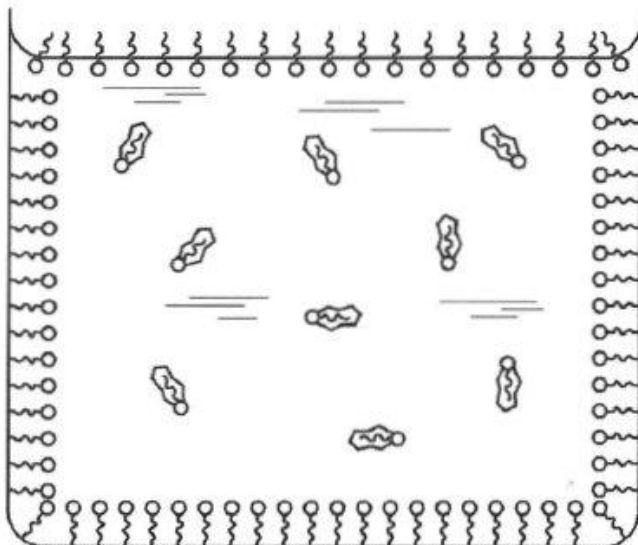
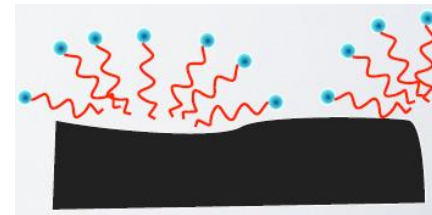
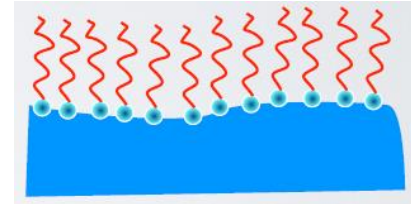
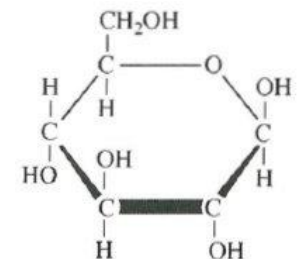


Fig. 3-16: Schematic of surfactant adsorption from aqueous solution, both to the air-water surface and the water-solid interface of the container wall. The "iceberg" of structure water surrounding the hydrophobic tails in solution is depicted.

glucose **not a surfactant** because hydrophilic hydroxyl groups are **not segregated** from the hydrocarbon structure



# Surfactants (III)

surfactants **strongly decrease**  $\sigma$ ,  
even @ low concentrations

$\sigma(c_{\text{surf}})$  curves remarkably **similar**  
for different surfactants

above a critical  $c_{\text{surf}}$ ,  $\sigma$  remains constant  
→ **critical micellar concentration**

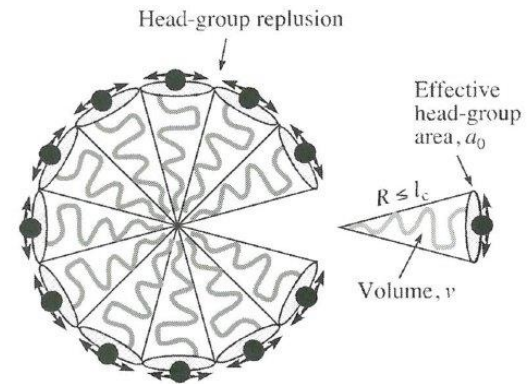
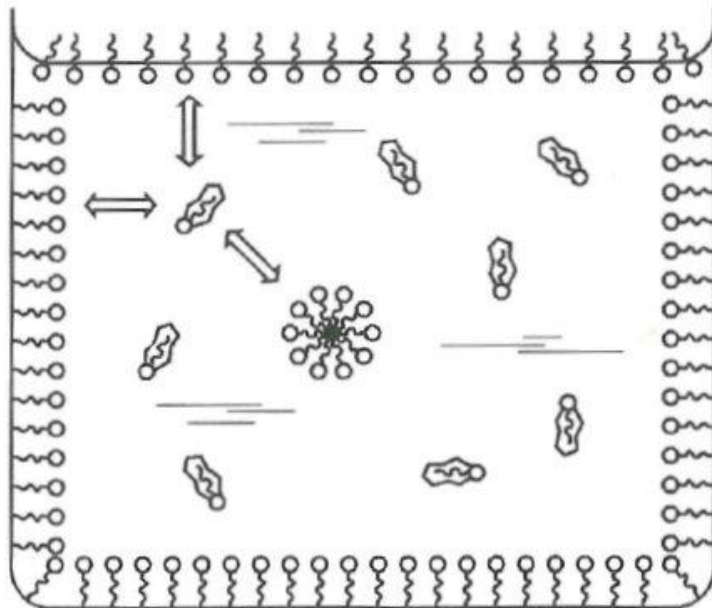
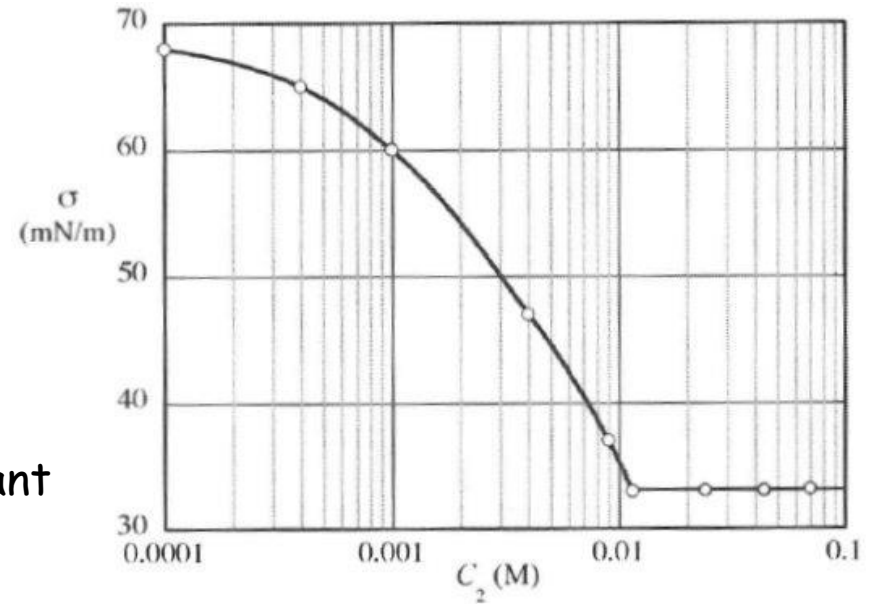


Fig. 3-31: Micelles as shape-filling structures, depend upon the size and shape characteristics of the monomer.